Property maps for abrasion resistance of materials

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Abstract

Using mechanics models of contacting surfaces under both normal and tangential loads, a mechanistic framework has been developed for assessing material resistance to initiation of abrasive damage. Solutions are presented for the critical loads to initiate yielding and cracking at blunt contacts, as well as those to attain a prescribed plastic penetration depth and cause cracking from within the plastic zone at sharp contacts. Material property groups that characterize abrasion resistance emerge from the models. Guided by these groups, illustrative property maps are constructed and used to make comparative assessments of a wide range of engineering materials.

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1. Introduction

Engineering materials are subjected to abrasion during contact with external bodies [1]. Despite its technological importance, the notion of abrasion resistance as a material property remains elusive. Three problems exist. (i) Abrasion is a multi-body process, involving interactions between at least two materials, usually with dissimilar properties, and, possibly, an intervening fluid. As such, abrasion is inherently a system property, rather than a material property. (ii) Abrasion occurs via various mechanisms, including plasticity and cracking, under single or multiple (fatigue) load cycles. It can also involve adhesion and subsequent decohesion, as well as oxidation. A consequence of multiple mechanisms is that there is no single metric that characterizes material performance. (iii) Even in seemingly simple geometric configurations (e.g. two flat contacting plates), the stress states in the contact regions are complex because of surface asperities. The stresses that govern abrasion are highly localized and decay rapidly with distance from the contact sites.

Notwithstanding these complexities, significant insight into abrasion resistance can be gleaned from analyses of the idealized contact scenarios illustrated in Fig. 1. These scenarios include yielding and cracking, under either normal load or combined normal/tangential loads, and with abrasive media creating either blunt or sharp contacts.

The principal objective of this paper is to present a mechanistic framework for assessment of abrasion resistance. The assessment is made on the basis of the normal and tangential loads needed to initiate abrasive damage during a single contact or sliding event. Several material property groups emerge from the analyses, each representing a metric of resistance to one potential abrasion mechanism. A secondary objective is to utilize the framework to compare the abrasion resistance of a broad range of engineering materials. For this purpose, property maps are constructed using coordinates that emerge from the property groups (in the spirit of Ashby [2]).

A complementary framework for depicting the mechanisms associated with contact loadings has been presented by Sharpe et al. [3]. This leads to the construction of property maps with coordinates of toughness/modulus and strength/modulus. By equating, in pairs, the loads for elastic and plastic deformation and onset of cracking, analytical formulae for the mechanism boundaries are obtained.
The present work extends that of Sharpe et al., allowing quantitative comparisons of material performance.

2. Contact models

2.1. Preliminaries

The abrasion process is modeled by a two-body contacting system. Without much loss of generality, the material of interest is treated as a flat plate and the abrading material as a solid of revolution with local curvature, 1/R. Distinction is made between two contact types: blunt contacts, characterized by large values of 1/R, and sharp contacts, wherein R → 0.

The fundamental mechanical properties of the plate material are Young's modulus, E, Poisson's ratio, ν, yield strength, σy, and fracture toughness, Ky. Three other (dependent) properties emerge: hardness, H ≈ 3σy, fracture energy, Gc = Kc²(1 − ν²)/E, and plane strain modulus, E' = E/(1 − ν²). The abrading material is assumed to remain elastic during contact so that its mechanical response can be fully characterized by its Young's modulus E' and Poisson's ratio ν', along with the plane strain modulus E'' = E'/(1 − ν'²).

In general, both normal and tangential loads, P and Q, act on the contact, the two being related by Q = μP where μ is the friction coefficient. The respective stress fields of normal and tangential loads are essentially independent of one another and can thus be linearly superposed. In most cases, the contact area is nearly circular, with radius a. Provided a ≪ R, the stress distributions are given by the well-known Hertzian solutions [1,4]. These distributions are combined with appropriate failure criteria to determine the corresponding failure loads. Since only single loading excursions are considered, the resulting loads represent threshold values below which the contact is purely elastic and hence abrasive damage should be small upon repeated loading.

For blunt contacts, two failure modes are considered: (i) onset of yielding (Fig. 1a and b); and (ii) formation of cracks prior to yield (Fig. 1c and d). When the contacts are sharp, yielding is unavoidable, even at low loads. Two additional criteria are used for assessment: (i) attainment of a critical plastic penetration depth (Fig. 1e), and (ii) formation of cracks within the plastic zone (Fig. 1f). Each criterion yields a dominant material property group that serves as a performance metric.

2.2. Blunt contacts: plastic deformation

The conditions for yielding are obtained by setting the maximum deviatoric stress in the vicinity of the contact equal to σy. In the absence of a tangential load (μ = 0), the maximum value is located at a depth of about a/2 beneath the surface (Fig. 1a). The normal load, P0, for yielding is given by [1,4]:

\[ \frac{P_0}{R^2} = C_0^1 \frac{\sigma_y^3}{E^3}, \]  

where \( C_0^1 = 21 \) and E is the composite modulus:

\[ \frac{1}{E} = \frac{1}{E'} + \frac{1}{E'}. \]  

Combining Eqs. (1) and (2) and taking H ≈ 3σy yields:

\[ \frac{P_0}{R^2} = C_1 \left( \frac{H^3}{E''} \right) \left( 1 + \frac{E'}{E} \right)^2 \]  

with \( C_1 \approx 0.8 \). Here the failure load, characterized by \( P_0/R^2 \), is proportional to the material property group \( H^3/E'' \) and to an elastic mismatch parameter, \((1 + E'/E)^2\). In many cases, E'' ≫ E, such that the mismatch parameter is close to unity and thus the corresponding dependence on E (outside of H²/E'') is weak.

Beyond \( P_0^y \), the plastic zone expands with increasing load. However, while contained within the sub-surface region, the plastic strains are small: comparable to the elastic strains of the surrounding material. Only when the plastic zone reaches the free surface does the plasticity begin to develop in full earnest, leaving a detectable permanent impression after unloading [3,5]. The critical load \( P_0^y \) for this transition is about 3\( P_0^y \).

The yield process changes when tangential loads are also present at the contact [1,4]. As the load ratio \( μ = Q/P \) increases, the location of maximum deviatoric stress moves closer to the contact surface and its magnitude increases. Beyond a critical value, \( μ ≈ 0.3 \), the maximum stress occurs on the surface at the edge of the contact circle (Fig. 1b). The normal load for yielding is again obtained by setting the deviatoric stress equal to σy. In the first domain (sub-surface yielding), the normalized yield load P0y/Py for ν = 0.3 is given by the empirical formula (Fig. 2):
tangential loads are present. However, the sensitivity of \( P_y \) to \( \mu \) is important in determining the dominant failure mode: a feature that becomes evident later, when comparisons are made between competing modes.

### 2.3. Blunt contacts: onset of cracking

In hard, brittle materials, contact stresses can lead to the formation of cone cracks, emanating from surface flaws just outside the contact circle, prior to yielding [7–9] (Fig. 1c). Such cracks penetrate to a depth comparable to the contact radius \( a \) and arrest. Once present, the cracks compromise the mechanical integrity of the near-surface region, making it susceptible to chipping during additional contact cycles. The onset of cracking thus represents the second failure mode.

In the absence of friction, the stress field on the contacting surface is axisymmetric. The maximum principal stress, \( \sigma_T \), occurs at the contact circle and acts in the radial direction. Its magnitude scales with the mean contact pressure \( p_m \) in accordance with \( \sigma_T = (0.5 - \nu)p_m \). Below the surface, \( \sigma_T \) diminishes rapidly with distance. To obtain a tractable (albeit approximate) solution for the stress intensity factor for downward extension of an incipient crack of length \( c_f \), the crack is assumed to be planar and oriented perpendicular to the free surface, i.e. cone curvature and the diverging crack front are neglected. Setting the resultant stress intensity factor equal to \( K_c \) yields the critical load \( P_c \) for full cone crack formation.\(^1\) The result is [7]:

\[
\frac{P_c}{R} = C_2 G_c \left[ 1 + \frac{E}{E_i} \right],
\]

where \( C_2 \) is a constant. Two features of Eq. (6a) are notable. (i) \( P_c \) scales with \( R \) (not \( R^2 \)), in accord with the empirically established Auerbach law [10]. The seeming inconsistency with a critical stress failure criterion is due to the large stress gradient in the sub-surface region. Because of the gradient, \( P_c \) is dictated by the final crack depth (proportional to \( a \)) and is independent of the initial flaw size, \( c_f \). (ii) The dominant material property group that governs the resistance to cracking is \( G_c \); the dependence on \( E \) via the elastic mismatch parameter is usually small.

For subsequent comparison with the loads for yield initiation, Eq. (6a) is re-expressed as

\[
\frac{P_c}{R^2} = C_2 G_c \left[ 1 + \frac{E}{E_i} \right].
\]

First-principles calculation of \( C_2 \) is subject to large uncertainty because of the assumptions that need to be made about crack shape and stress distributions. It is best determined through indentation experiments on materials with known mechanical properties (notably \( G_c \) and \( E \)) using indenters of varying tip radius. Experiments of this type on a wide range of materials yield \( C_2 \approx 9000 \) [10,11].

Friction dramatically alters the critical normal load (Fig. 1d). In this case, the maximum principal stress at the contact circle is [7]:

\[
\sigma_T = (1 + 15.5\mu)(0.5 - \nu)p_m.
\]

For typical values of \( \mu \), say 0.5, \( \sigma_T \) is about an order of magnitude greater than that for normal loading alone. Moreover, with friction, the tensile stress persists to a greater depth beneath the surface. One consequence of the latter is that the flaws behave in a manner similar to that in a homogeneously stressed body (at least initially), in the sense that crack extension occurs at a critical tensile stress, dependent on \( K_c \) and \( c_f \) via the Griffith relation. But the cracks arrest at a finite depth, comparable to \( a \), because of the diminishing stress field at larger distances from the surface. For even small friction coefficients, a reasonable approximation for the cracking load is [7]:

\[
\frac{P_c}{R^2} = C_3 \left[ \frac{1}{(1 + 15.5\mu) c_f^{3/2}} \right] \left[ \frac{K_c^3}{E^2(1 - 2\nu)^2} \right] \left[ 1 + \frac{E}{E_i} \right]^2,
\]

\(^1\) When pre-existing flaws are very small, say 0.01\( a \), a stable crack of intermediate length, about 0.1\( a \), can form at a lower load. Such cracks are rather benign in relation to the fully developed cone cracks, which typically penetrate to a depth of order \( a \). Consequently, only the latter are deemed to constitute failure initiation.
where \( C_3 \approx 140 \). In contrast to the case of normal loading (Eq. (6a)), \( P_c \), scales with \( R^2 \). Additionally, the governing material property group is now \( K_c^4/E^3(1-2\nu)^3 \).

2.4. Sharp contacts: plastic penetration

When the abrasive has sharp corners \( (R \to 0) \), yielding is unavoidable, since \( P_c^o \propto R^2 \). In this case, two complementary failure criteria are invoked: (i) that the indent depth attains a prescribed critical value, and (ii) a crack forms within the plastic zone. The former is addressed in this section and the latter in the next.

Analytical solutions of plastic penetration are obtained from the extensive literature on indentation mechanics [12]. For sharp indenters \( (R = 0) \), the residual penetration depth after unloading from a peak load \( P \) is given by:

\[
h_c = \sqrt{\frac{P}{C_4H}} \sqrt{\frac{2\pi H}{E}}, \tag{9}\]

where \( \varepsilon = 0.75 \) and \( C_4 \) is a function of the indent geometry; for Berkovich and Vickers indenters, \( C_4 = 24.5 \). The first term on the right-hand side of Eq. (9) is the contact depth at peak load \( P \) and the second term is the elastic recovery upon unloading. Rearranging Eq. (9), to bring out the pertinent property groups, yields:

\[
\frac{P}{h_c^2} = \frac{C_4}{1 - (C_4H/E)(1 + E/E')}^2, \tag{10}\]

where \( C_5 = \sqrt{\pi C_4}/2, C_5 = \sqrt{\pi C_4}/2 \approx 4 \). In most cases, the term in the denominator on the right side is dominated by \( H/E \). Since its value is small (typically \( \leq 10^{-2} \)), Eq. (10) can be approximated by:

\[
\frac{P}{h_c^2} \approx C_4H. \tag{11}\]

Resistance to plastic penetration is thus dominated by hardness.

2.5. Sharp contacts: cracking

Cracking at elastic-plastic contacts has been studied extensively for indenters with flat faces and sharp corners, e.g., Vickers, Berkovich, Knoop and cube corner [10] (Fig. 1f). Dimensional analysis shows that the stress intensity factor of an incipient crack at an indenter edge follows the scaling \( K \propto H^{d/2} \), where \( d \) is the diagonal length of the resultant indent and the hardness scales as \( H \propto P/d^2 \). Combining these results and eliminating \( d \), the critical load for crack initiation becomes:

\[
P_c = C_6 \frac{K_c^4}{H^2}. \tag{12}\]

The pertinent property group is \( K_c^4/H^3 \). From data of the type presented in Ref. [13], the inferred value of the proportionality constant is \( C_6 \approx 1.5 \times 10^4 \) for the Vickers indenter. For indenters with smaller included angles, such as the cube corner, \( C_6 \) is significantly lower, by as much as one or two orders of magnitude [13].

3. Property maps

The property groups from the preceding analyses have been used to identify useful graphical representations of property data. Here, two classes of property maps are presented. In the first, each failure mode is addressed separately. Selection of the coordinate axes is based on the pertinent property groups. In the second, two failure modes are considered concurrently, leading to a different selection of axes. In all cases, the property maps have been constructed using the Cambridge Engineering Selector software package (Granta Design Ltd., Cambridge, UK). Material abbreviations used on the maps are defined in Table 1.

For yield initiation at a blunt contact, the dominant property group is \( H/E \). Thus, the preferred axes of the property map are \( H \) and \( E \). When plotted logarithmically, data lying on a straight line of slope \( 2/3 \) represent materials with equivalent performance, when the abrasive is rigid. (The secondary dependence on \( E \) via the elastic mismatch parameter is addressed below.) Fig. 3a shows such a map. Superimposed is a family of lines of constant \( H^3/E^2 \) and hence constant \( P_c^o/R^2 \). The map reveals (unsurprisingly) that ceramics are superior to other engineering materials. Among the ceramics, SiC and B_4C are the best; the others, including Si_3N_4, AlN, WC, Al_2O_3 and toughened zirconia, exhibit lower critical loads, by a factor of 2–10. Interestingly, the range of material performance within this class of materials is rather narrow; changes in \( E \) are accompanied by corresponding changes in \( H \) such that \( H^3/E^2 \) does not vary a great deal. The map also shows that some engineering polymers (polyamides and PEEK, for instance) are competitive with the hardest of the metallic alloys, despite large differences in their respective values of \( E \) and \( H \).

Fig. 3b shows the same property data, but with contours of fixed \( P_c^o/R^2 \) for finite values of \( E \), ranging from 30 to 1000 GPa. In this case, stiffer materials are preferred at constant \( H^3/E^2 \). The performance benefits can be dramatic when \( E \) is comparable to \( E' \). For instance, from Eq. (3), increasing \( E/E' \) from 0.1 to 1 results in a 3-fold increase in the yield load. Otherwise, when \( E/E' < 0.1 \), the effects of \( E \) are negligible. One consequence (evident from inspection of Fig. 3b) is that hard metallic alloys become increas-

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<td>Material abbreviations</td>
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ingly superior to the hard polymers as the modulus of the abrasive is reduced. (These conclusions are predicated on the abrasive remaining elastic. Because compliant materials are also inherently soft, contact with a stiff substrate may lead to yielding in the abrasive before the plate.)

Minimal modifications are needed to account for effects of friction on yield. With friction, the property maps retain the same form, with the critical loads for each curve being reinterpreted in accordance with Eqs. (4) and (5).

To assess resistance to cracking prior to yielding (with friction), the property group $K_3^c/E^2(1 - 2v)^3$ (from Eq. (8)), suggests the use of coordinates $K/(1 - 2v)$ and $E$. For rigid abrasives, constant cracking loads are represented by lines with slope $2/3$ (Fig. 4a). For this failure mode, the material rankings are reversed: ceramics being the worst and metals and polymers being the best. However, there are some exceptions. Toughened zirconias, for example, are competitive with the hardest polymers and only slightly inferior to cast irons and high-carbon steels: a consequence of their high fracture toughness.

A third example of a map for a single failure mode is shown in Fig. 4b. From Eq. (12), the property group characterizing the resistance to cracking at a sharp contact is $K_4^c/H^3$. The preferred coordinates are $K_c$ and $H$ and constant cracking loads are represented by lines of slope $3/4$. In this context, soft metallic alloys are the best. Some polymers (PE and ABS, for instance) are competitive with the hard metals. Collectively, polymers and metals outperform ceramics by a large margin. (Note that the values of $K_4^c/H^3$ for the guidelines on Fig. 4b differ from one another by a factor of $10^4$.)

Complementary property maps have been constructed to depict resistance to two failure modes. For blunt contacts without friction, the preferred coordinates are $H^3/E^2$ and $G_c$. Critical loads for yielding are represented by a family of horizontal lines, each for a prescribed value of $H^3/E^2$, and those for cracking by vertical lines, each at a prescribed $G_c$. Since either mode constitutes failure, the lower load is used to characterize abrasion resistance. To display this result on the map, the vertical lines are truncated below the intersection point with their horizontal counterparts, and vice versa, thereby yielding a family of L-shaped curves, one for each failure load. Materials with property data above and to the right of such a curve have higher critical loads. An example is shown in Fig. 5a. For indenter radii $R \leq 1 \text{ mm}$, failure is dictated by yield initiation for almost all materials. Consequently, SiC and B$_4$C...
again emerge as the best materials; Si₃N₄, AlN, WC, Al₂O₃ and soda-lime glass are only slightly inferior (by a factor of 3–10). As R increases beyond 1 mm, the cracking load decreases, in accordance with the scaling $P_c/R^2 \propto 1/R$. This has the effect of translating the vertical branches of the failure loci to the right. For ceramics, this can lead to a transition in the governing failure mode, from yielding to cracking, as illustrated in Fig. 5a. In contrast, failure of all metals and polymers is dictated by yielding for virtually all values of $R$ (up to at least 1 m). Consequently, the performance rankings obtained from Fig. 3 apply.

A property map that accounts for friction is shown in Fig. 5b. The coordinate axes are $H/E^2$ (from Eqs. (3)–(5)) and $K_c/E^2(1-2v)^{3/2}$ (Eq. (8)). The resulting failure loci are again depicted by L-shaped curves: a horizontal component for cracking and a vertical one for yielding. In contrast to the previous case (no friction), the critical loads $P_c/R^2$ for yielding and cracking are independent of $R$, but decrease with increasing $\mu$. The cracking load also depends on the length $c_f$ of near-surface flaws (via Eq. (8)). For $c_f = 10$ µm (the value used in construction of Fig. 5b), polymers and metals generally outperform ceramics, especially...
For large friction coefficients, for instance, for $P_c/R^2 = 0.3$ MPa and $\mu = 0.3$, ceramics are unacceptable (except Si$_3$N$_4$ and zirconia), whereas many polymers and metals exhibit adequate performance. Increasing $c_f$ has the effect of translating the vertical branches to the right, eliminating more ceramic candidates at a prescribed load and making the polymers and metals even more attractive.

For sharp contacts, resistance to both plastic penetration and cracking is depicted using coordinates $H$ (from Eq. (11)) and $K_4^c/H^3$ (Eq. (12)). Once again, material performance is represented by a family of L-shaped curves (Fig. 5c). For sensible choices of $h_r (1–100 \mu m)$, failure of ceramics is always controlled by cracking, whereas that of metals and polymers is controlled by plastic penetration. Consequently, among the latter materials, those with high hardness (notably metals) are always preferred. But the ranking of ceramics and metals is not universal. Because of the strong sensitivity of the critical load to the penetration depth, notably $P_c/h_r^2$, ceramics emerge as the best materials for small values of $h_r$ and vice versa.

4. Conclusions

A mechanistic framework for assessing abrasion resistance of materials has been developed and implemented. Comparisons of material performance are most conveniently made using property maps, with coordinate axes selected from the formulae for critical loads to activate each failure mode. The most useful maps are those in Fig. 5. These cover both blunt and sharp contacts as well as the effects of friction.

Although no single material emerges as being superior for all loading scenarios, the comparisons yield important insights into material selection for abrasion resistance. For instance, for blunt contacts with small abrasive radii ($\leq 1$ mm) and no friction, damage initiation occurs by yielding in virtually all materials. Consequently, those with the highest values of $H^3/E^2$ are preferred. In this context, ceramics are the best (especially SiC and B$_4$C). High-strength metals such as Ti alloys and high-carbon steels rank next, with $H^3/E^2$ values about 1–2 orders of magnitude lower. Surprisingly, some polymers (PA and PEEK) fall in the same category. The remaining polymers and metals are less resistant, by another 1–2 orders of magnitude.

In the presence of friction, the rankings among metals and polymers remain the same; although friction lowers the load for yielding, it does not alter the scaling of the load with the pertinent group, notably $H^3/E^2$. Its effect is most pronounced in ceramics. Even for small values of $\mu$, onset of failure in ceramics is dictated by cracking, not yielding. Furthermore, the cracking load depends sensitively on $\mu$. For moderately high values, say $\mu \geq 0.3$, many metals and polymers outperform ceramics.

Among metals and polymers, selection of materials for resistance to sharp abrasives is dictated exclusively by hardness, with metals coming out on top. But the ranking of ceramics vs. other materials is less clear cut. For small values of $h_r$, ceramics are preferred, since the loads to initiate failure are usually higher. But when failure does occur, it involves crack formation, rather than just plastic penetration. This mode is likely to be more deleterious in the sense that it would lead to a greater reduction in abrasion resistance during subsequent contact events. Assessment of such effects would require analysis of repeated loadings at contact sites with prior plasticity and cracking.

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